Running Head: REVIEW OF INFANT NUMBER

Open questions and a proposal:

A critical review of the evidence on infant numerical abilities

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Abstract

Considerable research has investigated infants' numerical capacities. Studies in this domain have used procedures of habituation, head turn, violation of expectation, reaching, and crawling to ask what quantities infants discriminate and represent visually, auditorily as well as intermodally. The concensus view from these studies is that infants possess a numerical system that is amodal and aplicable to the quantification of any kind of entity and that this system is fundamentally separate from other systems that represent continuous magnitude. Although there is much evidence consistent with this view, there are also inconsistencies in the data. This paper provides a broad review of what we know, including the evidence suggesting systematic early knowledge as well as the peculiarities and gaps in the empirical findings with respect to the concensus view. We argue, from these inconsistencies, that the concensus view cannot be entirely correct. In light of the evidence, we propose a new hypothesis, the Signal Clarity hypothesis, that posits a developmental role for dimensions of continuous quantity within the discrete quantity system and calls for a broader research agenda that considers the covariation of discrete and continuous quantities not simply as a problem for experimental control but as information that developing infants may use to build more precise and robust representations of number.

Keywords: number, quantity discrimination, infant

1. Introduction

Considerable research suggests that numerical reasoning originates in a basic capacity that is independent of culture or language. When asked to discriminate, estimate, or transform quantities, human adult judgments are systematic without the use of counting or formal mathematical strategies. For small quantities, humans have shown exact judgments within the range of 1 to approximately 4 items (Kafman, Lord, Reese, & Volkmann, 1949; Mandler & Shebo, 1982; Taves, 1941). Large quantity judgments, although not exact, are systematically patterned across species: for human and nonhuman primates— as well as a large range of other animals including rats and pigeons— discrimination is subject to Weber's Law (Cordes, Gelman, Gallistel, & Whalen, 2001; Whalen, Gallistel, & Gelman, 1999, Brannon & Terrace, 1998; Meck & Church, 1983; Roberts & Mitchell, 1994). In the past three decades research has pursued the question of whether human infant numerical judgments show these same signature regularities. The consensus is that they do (Carey, 2009; Dehaene, 1997; Feigenson, Dehaene, Spelke, 2004); results from experiments using a variety of different methods show that infants discriminate, track, and transform quantities and do so in ways that resemble the behavioral patterns of adults and other animals in laboratory experiments (e.g., Cordes & Brannon, 2009b; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005).

Accordingly, the predominant view— and the starting point for many theories of numerical concepts— is that human infants have a capacity to represent discrete amounts (e.g., Carey, 2009; Cordes & Brannon, 2008; Feigenson, Dehaene, & Spelke, 2004; Spelke & Kinzer, 2007; Xu & Spelke, 2000). By this perspective, infants perceive, represent, and discriminate quantities using an evolutionarily ancient system – one that is specifically tuned to number. There is substantial evidence for this general conclusion. However, there are two additional theoretical ideas associated with this proposal. The first of these is that the evolutionarily ancient numerical system is fundamentally separate from other systems of magnitude discrimination and representation. The second is that the discrete number system is abstract and amodal, and thus not limited to one sensory modality but rather applicable to the quantification of *any* kind of entity (e.g., sights, sounds, actions, Lipton & Spelke, 2003, 2004; Wynn, 1996). An abstract and early discrete number system that is distinct from other forms of magnitude judgment is counter to the classic developmental theory of Piaget (1952), which proposes that the capacities observed in infancy— although the foundation of later numerical competence— are not initially specific to number. There are also contemporary researchers who suggest that a discrete number system may be built out of a more general magnitude system (see Mix, Huttenlocher, & Levine, 2002; Gebuis & Reynvoet, 2011; see also Lourenco & Longo, 2011 for related perspectives); but this is the minority view in the literature.

The claim that infant or adult perception of discrete quantity is in some way *separate* from the modality specific properties of the array including other dimensions of magnitude (such as the amount of visual spread in an array) is difficult to demonstrate empirically and is the source of complication for experimental methods. These complications are especially problematic in the infant literature given the necessary limits on the number of trial types and dependent measures. The fundamental problem is that discrete quantity in the environment is correlated with other stimulus dimensions; as the number of discrete elements in a set increases, other perceptual properties change as well, and although one might control one of these properties in any one experiment, all of them cannot be controlled simultaneously. These complexities in experimental control bring us to the core question motivating this review: The consensus view of an evolutionarily old, mechanistically distinct and developmentally early number system yields a set of clear predictions. Although many of these predictions are supported by empirical data, there are also key failures. How should the field understand these problematic results and how should we evaluate the consensus view in their light?

To address these questions, we first provide a comprehensive review of studies that investigate quantitative capacities in infants— many of which support the consensus view. We then take a closer look at the more problematic cases. Our conclusion is that the acceptance of the predominant view is not yet warranted and that these problematic cases might not be best viewed as noise that can be ignored but rather as the nonsinging canary in the coal mine— an indication that there is something amiss in our current understanding of early quantitative capacities. In the final section we propose a new theoretical framework that may more wholly account for the data: infants are highly sensitive to the statistical regularites in the environment; there are correlations between discrete quantity and other dimensions of magnitude, and these correlations support the development of internally-stable and finely-tuned quantity judgements. Our proposal is compatible with the idea of an evolutionarily and developmentally early number system, although it might require a modification in our conception of exactly *what* the evolutionarily early system *is* and may require us to abandon the assumption that the numerical system is completely segregated from other dimensions of continuous quantity representation or abstract at its onset. Whether our proposal or the current concensus view proves more correct in the end, our analysis also suggests the value of a shift in the research agenda—a shift away from the current emphases that rule out a role for stimulus properties other than number itself to a study of numerical cognition— and a study of the developmental changes in how nonsymbolic number is processed— that is in relation to the correlated dimensions of magnititude.

2. Current Research in Infants' Numerical Capacities: Methods and Findings

2.1. Infants' numerical discriminations: detecting differences visually and auditorily

The first studies of infant numerical abilities and many that have followed in the past three decades have tested discrimination of nonsymbolic quantities using habituation and familiarization procedures. The studies have asked the empirical question of whether infants can tell the difference among varying numerosities of geometric figures, pictures, events, or sounds. In a seminal study, Starkey and Cooper (1980) habituated 3-22 week old infants to visual displays of various numerosities (e.g., 2 or 3 black dots) and then presented the infants novel quantities. In testing, infants dishabituated to a change in number; infants habituated to 2 dots dishabituated to 3 and vice versa, indicating that they detected the change in quantities. Studies that followed found similar results. Antell & Keating (1983) found the same result in a replication of this experiment with neonates. In another classic study, Strauss and Curtis (1981) habituated infants to arrays of pictures of everyday items that varied in their quantities. In this experiment, 10-12 month old infants also discriminated 2 from 3 items as well as 3 from 4.

Since the original Starkey and Cooper (1980) study, many other experiments have used this same general procedure to investigate infants' abilities. A list of visual numerical discrimination studies using the habituation or familiarization procedure is found in Table 1. The studies in the table are organized according to the quantities tested and are arranged in the general ascending order of those quantities with columns indicating whether infants discriminated the quantities. The accumulation of data, as can be seen in Table 1, has formed a picture of a capacity with signature traits. One signature trait is the ratio limit of large number discrimination; infants discriminate large quantities only approximately rather than based on absolute values, detecting differences in accordance with Weber's Law. For example, infants at 6 months discriminate differences at a 1:2 ratio; they discriminate 8 from 16 items and 16 from

32, but not 16 from 24 (e.g., Brannon, Abbott, & Lutz, 2004; Cordes & Brannon, 2009b; Lipton & Spelke, 2003; Xu & Spelke, 2000). The Weber fraction decreases with age; infants discriminate at 2:3 ratios by 10 months (Xu & Arriaga, 2007).

The second signature trait of numerical discrimination is that, although larger quantities are subject to Weber's Law, smaller quantities are not. It has been suggested that smaller quantities are more precisely apprehended (see Carey, 2009 for a review). Six-month-old infants in some cases have successfully discriminated 2 from 3, despite not being able to discriminate a 2:3 ratio difference for larger numbers (e.g., Bijeljacbabic, Bertoncini, & Mehler, 1993; Cordes & Brannon, 2009b; Starkey & Cooper, 1980; Xu & Spelke, 2000). A further related phenomenon of early discrimination is what appears to be a *divide* between the processes infants use to quantify small and large numbers. Although infants can discriminate large numbers of sufficient ratio differences and sometimes discriminate small numbers more precisely, they seem unable to directly compare quantities from large and small sets. Infants have failed in discrimination of 2 and 4 and seemingly do not discriminate 3 and 6—despite the ability to discriminate larger quantities at a 1:2 ratio (Cordes & Brannon, 2009a; Xu, 2003). These findings have led some researchers to hypothesize that there are two separate systems for quantifying small and large numerosities, and early on the processes for the two systems may be so fundamentally different that quantities of each set size cannot be compared (Feigenson, Dehaene, & Spelke, 2004).

Although early studies only investigated the visual discrimination of two-dimensional visual arrays, additional research has shown that infants successfully discriminate numbers of events (e.g., puppet jumps) as well as auditory tones (Lipton & Spelke, 2003, 2004; Wynn, 1996). Table 2 lists studies that have tested auditory discrimination of number. Importantly, some of these investigations have reported that infants' performance in these tasks shows the same signature traits as in visual discrimination studies: large quantities are subject to ratio limits; small quantities are not, and there may be a divide between processes for discriminating each (see especially Lipton & Spelke, 2003, 2004). Also noteworthy is that, although the majority of visual and auditory studies have used a familiarization or habituation procedure, more recently researchers have used change detection to test infant discrimination (Libertus & Brannon, 2010; Starr, Libertus, & Brannon, in press) as well as neuroimaging techniques (e.g., Hyde & Spelke, 2011; Libertus, Pruitt, Woldorff & Brannon, 2009) and these may also show similar patterns of quantity processing and discrimination.

2.2 Infants' small set tracking: visual working memory, object representation, and knowledge of more

Spurred by adult research as well as the just described infant results, researchers have further investigated the possibility of a separate system used to represent small quantities. Many studies investigating small set quantification have use procedures that incorporate real objects in dynamic events. In these procedures, infants are shown small quantities of toys, balls, or crackers being hidden in boxes, buckets or behind occluders. Infants' knowledge of the quantity of items is then tested either as measured by their reaching time or their choices for a particular hidden quantity. There are two main procedures in this domain: crawling and manual search. Tables 3 and 4 provide a list of studies, the quantities compared, and the results using these two methods.

Both crawling and manual search procedures use motor behavior as the dependent measure. In crawling procedures, infants are shown crackers placed into two different buckets. Infants are then allowed to crawl to one of the two buckets. In this procedure, infants reliably crawl towards the bucket in which they have seen a greater quantity of crackers hidden. For example, infants reliably choose the bucket in which 2 crackers are hidden over the bucket where only 1 cracker is hidden (Feigenson, Carey, & Hauser, 2002). Manual search studies have shown similar results. In these studies infants are shown some quantity of items (e.g., 2 balls) being placed into a box. They then observe an experimenter pulling out a quantity (e.g., 1 ball) and are then allowed to reach for the remaining item(s). Reaching time is compared to a baseline for the same infant's reaching after seeing one item hidden (when the expected remaining quantity is 1) as well as a condition in which they see 1 item hidden and retrieved (when the expected quantity remaining is none—an empty box). In such studies, infants show significant differences in their reaching time—they reach longer when the expected quantity is 1 or 2 items than when the expected quantity is none (Feigenson & Carey, 2003, 2005). These results are interpreted as evidence that infants track the precise number of objects for these small quantities of 1, 2, and 3.

Results using these same procedures also support the hypothesis that there may be a division between small and large numbers in the early cognitive system. For example, infants in these procedures show memory and tracking for 1, 2, and 3 balls; however, when infants are shown 4 balls hidden in a box, their reaching time is indiscriminate from the amount of time spent reaching for 1 ball. This lack of increased search time for 4 items has been interpreted in terms of a limit on the number of objects that can be represented; once the set size exceeds a limit of 3, infants cannot robustly represent and track discrete items (Feigenson & Carey, 2005). Consistent with this interpretation, studies using the crawling procedure have found that infants reliably crawl towards the bucket with a greater number of hidden crackers when quantities are between 1 and 2, 2 and 3, 1 and 3 but not 1 and 4 or 2 and 4 (Feigenson & Carey, 2005;

Feigenson, Carey, & Hauser, 2002). These findings suggest two main ideas. First, small quantities appear to be more precisely tracked but there is a limit on the number that can be represented. This number (3-4 items) coincides with research with adults showing a similar limit on precise quantification and object tracking (e.g., Kafman, Lord, Reese, & Volkmann, 1949; Taves, 1941; Trick & Pylyshyn, 1994). Second, and given the similarity to adult capacities, these infant behaviors may come from the same object-based system that are responsible for adult precision with small quantities (Feigenson et al., 2004).

It is important to note that while these search task studies show clear evidence of capacities to track and quantify small sets of items, infants' performances require more than mere discrimination of quantities. Infants are required to remember amounts *and* remember their locations, and then to base motivated behavior on this knowledge. These tasks are therefore also dependent upon the development of visual working memory, object representation, and knowledge of "more"—not just quantity discrimination. Related and also important to note is that these studies have been conducted with infants relatively older (10-14 months) than those who participated in the previously described discrimination studies using habituation and familiarization procedures (birth-8 months). Crawling and manual search tasks are therefore more demanding than simple discrimination and one might argue from this that they are less suited to answering the more specific question of whether there is a separate system in very young infants for quantifying small set.

Table 1.
NUMBER DISCRIMINATION Visual Discrimination, *Habituation/ Familiarization Procedures*

Study	Age	Quantities Tested	Dishabituated to novel numerosity?	Continuous extent variables controlled?	Dishabituated to novel continuous extent?	Dishabituated to familiar numerosity?	Difference in looking during test?
Feigenson, Carey, & Spelke (2002) Experiments 1-4	6-7 mos	1 vs 2	**No-3**	Yes (Exp 1-3) Yes‡ (Exp 4)	Yes (Exp 1-2) No (Exp 3) NA (Exp 4)	No (Exp 1, 3, 4) Yes (Exp 2)	Yes (Exp 1-2) No (Exp 3-4)
Xu, Spelke, & Goddard (2005) Experiment 4	5-6 mos	1 vs 2	No*	Yes‡	NA	No*	**No**
Feigenson (2005)	6-7 mos	1 vs 2	**Yes-3**	Yes‡	NA	No	No
Cordes & Brannon (2009a)	6-7 mos	1 vs 4	**Yes-1**	Yes‡	NA	No	**Yes**
Antell & Keating (1983)	newborn	2 vs 3	**Yes-1, 2**	No	Yes-1, 2	NA	*
Strauss & Curtis (1981)	10-12 mos	2 vs 3	*	Yes‡ *	NA	*	**Yes**
Starkley & Cooper (1980)	3-22 wks	2 vs 3	**Yes-1, 2**	No	Yes-1, 2	NA	NA
Van Loosbroek & Smitsman (1990)	5 mos	2 vs 3	**No-1**	No	*	*	*
Van Loosbroek & Smitsman (1990)	8 and 13 mos	2 vs 3	**Yes-1**	No	*	*	*

Wynn (1996)	5-6 mos	2 vs 3	Yes* (Exp 1) No* (Exp 2)	Yes	NA	No	**Yes**
Clearfield (2004)	5-7 mos	2 vs 3	*	Yes (Exp 1) Yes‡ (Exp 2-3)	*	Yes-2* (Exp 2)	Yes (Exp 1) No (Exp 2) **Yes (Exp 3)**
Clearfield & Mix (1999)	6-7 mos	2 vs 3	**No-2**	Yes	Yes-2	Yes-2	Yes*
Clearfield & Mix (2001)	6-7 mos	2 vs 3	**No-1**	Yes	Yes-1	Yes-1	**Yes**
Feigenson, Carey, & Spelke (2002) Experiment 5	6-7 mos	2 vs 3	**No-3**	Yes‡	NA	No	No
Starkey, Spelke, & Gelman (1990)	6-9 mos	2 vs 3	No*	Yes‡	NA	No	**Yes**
Cordes & Brannon (2009b)	6-7 mos	2 vs 3	**Yes-1**	Yes	Yes-1	Yes-1	No
Brez & Colombo (2011)	5-6 mos	2 vs 3	*	Yes‡	NA	No*	**No**
Xu (2003)	5-6 mos	2 vs 4	*	Yes‡	NA	*	**No**
Cordes & Brannon (2009a)	6-7 mos	2 vs 4	Yes-1	Yes‡	NA	Yes-1	**No**
Wood & Spelke (2005b)	5-6 mos	2 vs 4	No*	Yes‡	NA	No*	**No**
Wynn, Bloom, & Chiang (2002)	4-5 mos	2 vs 4	Yes*	No*	Yes*	Yes*	**Yes**
Cordes & Brannon (2009b)	6-7 mos	2 vs 8	**Yes-1**	Yes	No-1	No	Yes

Cordes & Brannon (2009a)	6-7 mos	2 vs 8	Yes-1	Yes‡	NA	No	**Yes**
Strauss & Curtis (1981)	10-12 mos	3 vs 4	*	Yes‡	NA	*	**Yes**
Van Loosbroek & Smitsman (1990)	5 mos	3 vs 4	**No**	No	*	*	*
Van Loosbroek & Smitsman (1990)	8 and 13 mos	3 vs 4	**Yes-1**	No	*	*	*
Cordes & Brannon (2009a)	6-7 mos	3 vs 6	Yes-1	Yes‡	NA	No	**No**
Strauss & Curtis (1981)	10-12 mos	4 vs 5	*	Yes‡	NA	*	**No**
Treiber & Wilcox (1984)	4 mos	4 vs 5	**Yes**	Yes‡	NA	No	NA
Van Loosbroek & Smitsman (1990)	5 mos	4 vs 5	**No-1**	No	*	*	*
Van Loosbroek & Smitsman (1990)	8 and 13 mos	4 vs 5	**Yes-1**	No	*	*	*
Starkley & Cooper (1980)	3-22 wks	4 vs 6	**No-1, 2**	No	No-1, 2	NA	NA
Antell & Keating (1983)	newborn	4 vs 6	**No-1, 2**	No	Yes	NA	NA
Wood & Spelke (2005b)	5-6 mos	4 vs 6	No*	Yes‡	NA	No*	**No**

Wood & Spelke (2005b)	8-9 mos	4 vs 6	Yes*	Yes‡	NA	No*	**Yes**
Xu (2003)	6-7 mos	4 vs 8	*	Yes‡	NA	*	**No**
Wood & Spelke (2005b)	5-6 mos	4 vs 8	No*	Yes‡	NA	No*	**Yes**
Wood & Spelke (2005a)	4-5 mos	4 vs 8	Yes*	Yes‡	NA	No*	**Yes (2 sec display)**
Wood & Spelke (2005a)	4-5 mos	8 vs 16	Yes*	Yes‡	NA	No*	**Yes**
Wood & Spelke (2005a)	4-5 mos	4 vs 16	Yes*	Yes‡	NA	No*	**Yes**
Xu & Arriaga (2007)	9-10 mos	8 vs 10	*	Yes‡	NA	*	**No**
Xu & Arriaga (2007)	9-10 mos	8 vs 12	*	Yes‡	NA	*	**Yes**
Xu & Spelke (2000)	6-7 mos	8 vs 12	*	Yes‡	NA	*	**No**
Jordan, Suanda, & Brannon (2008) auditory-visual	5-6 mos	8 vs 12	**Yes-1**	Yes‡	NA	No	**Yes**
Clearfield (2005) Experiment 2	6-7 mos	8 vs 12	**No-1**	Yes	**Yes**	Yes	Yes
Cordes & Brannon (2009b)	6-7 mos	8 vs 16	**Yes-1**	Yes	No-1	No-1	No
Clearfield (2005) Experiment 1	6-7 mos	8 vs 16	**No-1**	Yes	**Yes**	Yes	**Yes**

Xu & Spelke (2000)	6-7 mos	8 vs 16	*	Yes‡	NA	*	**Yes**
Brannon, Abbot, & Lutz (2004)	5-6 mos	8 vs 16	No-1	Yes‡	NA	No	**Yes**
Cordes & Brannon (2008)	6-7 mos	7 vs 21	No*	Yes‡	NA	No*	**Yes**
Xu, Spelke, & Goddard (2005)	5-6 mos	16 vs 24	No*	Yes‡	NA	No*	**No**
Xu, Spelke, & Goddard (2005)	5-6 mos	16 vs 32	No*	Yes‡	NA	No*	**Yes**

This table is a list of studies conducted to investigate infants' visual discrimination of quantities using habituation or familiarization procedures and looking time as the dependent measure. Column 1 indicates the published paper from which the study comes. Column 2 indicates the ages tested. Column 3 indicates the quantities tested. Columns 4 through 8 address questions related to discrimination of the quantities. Column 4 indicates whether infants dishabituated to the novel numerical quantity. Column 5 indicates whether continuous extent was controlled at a 1:2 ratio level for surface area or cumulative contour. A Yes in this column indicates that continuous extent was controlled by holding it constant across habiutation/familiarization and testing (Clearfield & Mix control method). A Yes‡ indicates that continuous extent was controlled by varying it widely during habituation/familiarization (Xu & Spelke control method). Columns 6 indicates whether there was dishabituation to the novel continuous extent. This column only applies to those studies that either confound number and continuous extent or that directly test continuous extent dissociated from number. Column 7 indicates whether there was dishabituation to a familiar numerosity. Because studies do not all define dishabituation in the same way, three different measures for dishabituation in Columns 4, 6 and 7 are marked in the table as a number 1, 2, or 3 immediately after the response in the columns (e.g., Yes-1). The numbers correspond to the three following kinds of measures. Measure 1: The mean of the last two or three habituation trials is compared to the mean of the test trials (usually 2 or 3 trials for a condition). Measure 2: The last habituation trial is compared to the first test trial. Measure 3: The mean of the last three habituation trials is compared to the first test trial. Column 8 indicates whether there was a difference in looking times for the test stimuli (familiar and novel numerosities). Because many studies do not report statistical analyses or measures needed for responses to all columns, responses have been approximated by interpreting graphs and means. An approximation is denoted by an * in the table. In the cases in which dishabituation has not been reported, it has been approximated as a 1.5 fold increase in looking from the last habituation trial to the first test trial. In cases in which a difference in looking during testing has not been reported, it has been approximated as at least a 2 fold difference in looking times. In the cases in which no information (no graph *or* looking times *or* relevant statistical analysis) is provided, a * is accompanied by no response. Bolded responses throughout the table indicate those responses researchers used to conclude whether or not infants detected the quantity change. A Yes in the last column typically indicates a preference for the novel number except for Clearfield and Mix (1999, 2001) and Clearfield (2004) in which the preference was for the novel continuous extent.

Table 2.
NUMBER DISCRIMINATION Auditory Discriminations*, Sucking, Head Turn, Habituation Procedures*

Study	Age	Quantities Tested	Discriminated?	Continuous Variables Controlled?
Bijeljac-babic, Bertoncini, & Mehler (1993)	4 days	2 vs 3	Yes	Yes‡
Lipton & Spelke (2004)	8-9 mos	2 vs 3	No	Yes‡
Lipton & Spelke (2004)	5-6 mos	2 vs 4	No	Yes‡
Lipton & Spelke (2004)	8-9 mos	4 vs 5	No	Yes‡
Lipton & Spelke (2004)	5-6 mos	4 vs 6	No	Yes‡
Lipton & Spelke (2004)	8-9 mos	4 vs 6	Yes	Yes‡
Lipton & Spelke (2004)	5-6 mos	4 vs 8	Yes	Yes‡
Lipton & Spelke (2003)	8-9 mos	8 vs 10	No	Yes‡
Lipton & Spelke (2003)	5-6 mos	8 vs 12	No	Yes‡
Lipton & Spelke (2003)	8-9 mos	8 vs 12	Yes	Yes‡
Lipton & Spelke (2003)	5-6 mos	8 vs 16	Yes	Yes‡
vanMarle & Wynn (2009)	5-7 mos	2 vs 4	Yes	Yes‡
vanMarle & Wynn (2009)	6-7 mos	2 vs 3	No	Yes‡

This table is a list of studies conducted to investigate infants' auditory quantity discriminations. Many of these studies use a head turn procedure; the first study uses a sucking procedure; the last two studies use a habituation procedure. The structure of the table is similar to the Table 1. Column 4 indicates whether or not infants discriminated the quantities. Column 5 indicates whether or not continuous variables such as total sound duration and interstimulus intervals were controlled. Responses in this column accompanied by a ‡ indicate that these variables were varied during familiarization as a control.

Table 3.
SMALL NUMBER SET TRACKING Visual object tracking, *Manual Search Procedure*

Study	Age	Quantities Tested	Discrimination?	Continuous Variables Controlled?
Van de Walle, Carey, & Prevor (1997)	10 and 12 mos	1 vs 2	Yes	No
Feigenson & Carey (2003, 2005)	12 and 14 mos	1 vs 2	Yes	Yes*
Feigenson & Carey (2005)	12 mos	1 vs 3	Yes	No
Feigenson & Carey (2005)	10 and 12 mos	1 vs 4	No	No
Feigenson & Carey (2003, 2005)	12 and 14 mos	2 vs 3	Yes	No
Feigenson & Carey (2003)	12 and14 mos	2 vs 4	No	No
Feigenson & Halberda (2004)	14 mos	2 vs 4	Yes	No

This table is a list of relevant studies on small quantity set tracking as investigated through manual search. Column 4 indicates whether infants discriminated based on differential reaching to the hidden quantities use in this procedure. Column 5 indicates whether continuous variables of surface area or overall amount were controlled.

Table 4.
OBJECT SET TRACKING Visual tracking and discrimination, *Crawling Preference Procedure*

Study	Age	Quantities Tested	Preferred Greater Numerical Quantity?	Continuous Variables Controlled?
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	1 vs 2	Yes	No
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	1 vs 2	No	Yes
Cheries, Mitroff, Wynn & Scholl (2008)	10-12 mos	1 vs 2	Yes, when crackers are visible	No
VanMarle & Wynn (2011) Experiment 1a	10 and 12 mos	1 v 2	Yes	No
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	2 vs 3	Yes	No
Feigenson & Carey (2005)	10 and 12 mos	0 vs 4	Yes	
Feigenson & Carey (2005)	10 and 12 mos	1 vs 4	No	No
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	2 vs 4	No	No
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	3 vs 4	No	No
Feigenson, Carey, & Hauser (2002)	10 and 12 mos	3 vs 6	Yes, when crackers are visible	No
VanMarle & Wynn (2011) Experiment 1b	10-12 mos	5 v 10	Yes	No
VanMarle & Wynn (2011) Experiment 2 and 3a	10-12 mos	5 v 10	No	Yes

VanMarle & Wynn (2011) Experiment 3b	14 mos	5 v 10	Yes	No

This table is a list of relevant studies that have been conducted to investigate infants' small number and quantity tracking using the crawling procedure. Stimuli used to test quantities for these studies are usually crackers. Column 4 indicates whether or not infants preferred the greater quantity of crackers. A Yes in the column may be interpreted as infants' capacity to discriminate the quantities. Column 5 indicates whether continuous variables of surface area were controlled.

Table 5.
ARITHMETIC TRANSFORMATIONS Visual tracking, *Violation of Expectation Procedure*

Study	Age	Transformation Tested	Longer looking for Unexpected Number Transformation?	Continuous Variables Controlled?	Longer Looking for Unexpected Area Transformation?
Wynn (1992)	4-5 mos	1+1	Yes	No	Yes
Wynn (1992)	4-5 mos	2-1	Yes	No	Yes
Simon, Hespos, & Rochat (1995)	3-5 mos	1+1	Yes	No	Yes
Simon, Hespos, & Rochat (1995)	3-5 mos	2-1	Yes	No	Yes
Feigenson, Carey, & Spelke (2002), Experiements 6 & 7	6-7 mos	1+1	No	Yes	Yes
Feigenson, Carey, & Spelke (2002), Experiements 6 & 7	6-7 mos	2-1	No	Yes	Yes
Uller, Carey, Huntley-Fenner, & Klatt (1999)	8 and 10 mos	1+1	Yes	No	Yes
McCrink & Wynn (2004)	8-9 month olds	5+10	Yes	Yes	NA
McCrink & Wynn (2004)	8-9 month olds	10-5	Yes	Yes	NA

Gao, Levine, & Huttenlocher (2000)	6-7 mos	¼ + ½ = ¾	NA	NA	Yes

This table is list of relevant studies investigating infants' capacities to respond to transformations of quantity. Stimuli used for most of the studies in this list were three-dimensional dolls. One study (Gao, Levine, & Huttenlocher, 2000) investigated continuous quantity transformation and used liquid in a container as the stimuli. The transformation tested is in Column 3. Column 4 indicates whether infants looked longer at the unexpected outcome to the transformation (for example, looked longer to a scenario of 1+1=1). For the one study investigating continuous quantity transformations, the relevant information for discrimination is in Column 6.

Table 6.
ORDINAL RELATIONS Visual discrimination*, Habituation Procedure*

Study	Age	Relation Tested	Dishabituated to Novel Ordinal Relation?	Difference in looking during testing phase?
Macchi Cassia, Picozzi, Girelli, & de Hevia (2012)	4 mos	Ascending	No	**Yes**
Picozzi, Dolores de Hevia, Girelli, & Cassia (2010)	7 mos	Ascending	No	**Yes**
Brannon (2002)	8-9 mos	Ascending	**No**	**No**
Suanda, Tompson, & Brannon (2008)	8-9 mos	Ascending	**No**	**No**
Brannon (2002)	10-11 mos	Ascending	**Yes**	**Yes**
Suanda, Tompson, & Brannon (2008)	10-11 mos	Ascending	**Yes**	**Yes**
Cooper (1984)	10-12 mos	Ascending	No*	*
Cooper (1984)	14-16 mos	Ascending	Yes*	*
Macchi Cassia, Picozzi, Girelli, & de Hevia (2012)	4 mos	Descending	No	**No**
Picozzi, Dolores de hevia, Girelli, & Cassia (2010)	7 mos	Descending	No	**Yes**
Brannon (2002)	8-9 mos	Descending	**No**	**No**

Suanda, Tompson, & Brannon (2008)	8-9 mos	Descending	**No**	**No**
Brannon (2002)	10-11 mos	Descending	**Yes**	**Yes**
Suanda, Tompson, & Brannon (2008)	10- 11 mos	Descending	**Yes**	**Yes**
Cooper (1984)	10-12 mos	Descending	No*	*
Cooper (1984)	14-16 mos	Descending	Yes*	*

This table is a list of the studies that have tested infants' detection of ordinal relations. Column 4 indicates whether infants dishabituated to the change in ordinal relation from habituation to testing. A Yes in this column may be interpreted as infants' ability to discriminate and respond to a relation among various visual quantities.

Table 7.
INTERMODAL NUMBER MATCHING *Audio-Visual Preferential Looking and Violation of Expectation Procedures*

Study	Age	Quantities Tested	Preferred Matching Visual (for preferential looking studies)	Longer looking to Unexpected (for violation of expectation)
Starkey, Gelman, Spelke (1983, 1990)	6-8 mos	2 vs 3	Yes	NA
Moore, Benenson, Reznick, Peterson, & Kagan (1987)	6-8 mos	2 vs 3	No	NA
Mix, Levine, & Huttenlocher (1997)	6-8 mos	2 vs 3	No	NA
Kobayashi, Hiraki, & Hasegawa (2005)	5-6 mos	2 vs 3	NA	Yes
Jordan & Brannon (2006)	6-7 mos	2 vs 3	Yes	NA
Feron, Gentaz, & Streri (2006)	5 mos	2 vs 3	No (discrimination concluded)	NA
Izard, Sann, Spelke, & Streri, (2009)	newborns	4 vs 8	Yes	NA
Izard, Sann, Spelke, & Streri, (2009)	newborns	4 vs 12	Yes	NA
Izard, Sann, Spelke, & Streri, (2009)	newborns	6 vs 18	Yes	NA

This table is a list of studies that have been conducted to investigate infants' intermodal matching of quantities. Most of the studies used preferential looking procedures. One study used a violation of expectation procedure. Columns 4 indicates—for preferential looking studies— whether infants preferred the visual quantity that matched the auditory or tactile quantity. Column 5 indicates—for the one violation of expectation study—whether infants looked longer at the unexpected outcome of a visual-audio pairing.

Table 8.
CONTINUOUS EXTENT DISRIMINTATION Visual and Audio-Visual Discrimination*, Habituation/Familiarization Procedures*

Study	Age	Quantities Tested	Dishabituated to novel continuous extent?	Number Variables Controlled?	Dishabituated to novel numerosity?	Dishabituated to familiar continuous extent?	Difference in looking during test
Brannon, Suanda, & Libertus (2007)	9-10 mos	1.3 fold time duration change 3:4	**No-1**	Yes	NA	No	**No**
Brannon, Lutz, & Cordes (2006)	5-6 mos	1.5 fold area change 2:3	*	Yes	NA	*	**No**
vanMarle & Wynn (2006) Visual and Auditory	5-6 mos	1.5 fold time duration change 2:3	*	NA	NA	*	**No**
Brannon, Suanda, & Libertus (2007)	9-10 mos	1.5 fold time duration change 2:3	**Yes-1**	Yes	NA	No	**Yes**
Brannon, Lutz, & Cordes (2006)	5-6 mos	2 fold area change 1:2	*	Yes	NA	*	**Yes**
Brannon, Abbott, & Lutz (2004)	5-6 mos	2 fold area change 1:2	No-1	Yes‡	NA	No-1	**No**
vanMarle & Wynn (2006) Visual and Auditory	5-6 mos	2 fold time duration change 1:2	Yes, marginal significance reported	NA	NA	*	**Yes**

Brannon, Lutz, & Cordes (2006)	5-6 mos	3 fold area change 1:3	*	Yes	NA	*	**Yes**
Cordes & Brannon (2008)	5-6 mos	3 fold area change 1:3 (over small number set)	No-1	Yes‡	No	No-1	**No**
Cordes & Brannon (2008)	5-6 mos	3 fold area change 1:3 (over large number set)	Yes-1	Yes‡	Yes	Yes-1	**No**
Gao, Levine, & Huttenlocher (2000)	6-7 mos	3 fold mass change 1:3	Yes*	Yes	NA	No*	**Yes**
Brannon, Lutz, & Cordes (2006)	5-6 mos	4 fold area change 1:4	*	Yes	NA	*	**Yes**
Cordes & Brannon (2008)	5-6 mos	4 fold area change 1:4 (over small number set)	**Yes-1**	Yes‡	No*	No-1	**Yes**
Cordes & Brannon (2008)	5-6 mos	4 fold area change 1:4	No*	Yes‡	No*	No*	**Yes**
Hespos, Dora, Rips, & Christie (2012) Experiment 1	3-10 mos	4 fold area change 1:4	Yes* (7 mos old only)	Yes	*	*	**Yes**

Hespos, Dora, Rips, & Christie (2012) Experiment 2	7-13 mos	3 fold area change 1:3	No*	Yes	*	*	**Yes** (females only)
Cordes & Brannon (2011) Experiment 1a-2a	5-6 mos	3 fold item size change 1:3	*	Yes‡ (Exp1a) Yes (Exp 2a)	*	*	**Yes**
Cordes & Brannon (2011) Experiment 1b-2b	5-6 mos	4 fold item size change 1:4	*	Yes‡ (Exp 1b) Yes (Exp2b)	*	*	**Yes**

This table is a list of studies that have investigated infants' discriminations of continuous extent variables such as surface area and time duration. Column 4 indicates whether infants detected the continuous extent change that was tested. Column 5 indicates whether number was controlled. A Yes response in Column 5 indicates that researchers held the number constant across habituation and testing. A Yes‡ indicates that number was varied while the continuous extent dimension remained constant in habituation/familiarization. Column 6 indicates whether or not infants dishabituated to novel numerosities at testing. This column is only for those studies that varied number across habituation and testing in order to control number. (Thus, answering this question may only apply for studies that have a Yes‡ in Column 5.) Column 7 indicates whether or not there was a difference in looking times *during testing trials alone* between the novel and familiar quantity. Bolded responses throughout the table indicate those responses researchers used to concludes whether or not infants detected the quantity change.

2.3 Infants' calculations and ordinal relations

The ability to discriminate small sets may also underlie and therefore be evident in infants' capacity to represent transformation events (addition and subtraction). Studies that test infants' transformation abilities have generally used violation of expectation procedures. A list of these studies may be seen in Table 5. Wynn (1996) conducted one of the first transformation studies with young infants. In that study, 4-5 month old infants were shown addition and subtraction events using small sets of Mickey Mouse dolls. For example, infants watched as one or two dolls were hidden behind an occluder. The infants then watched as a hand reached behind the ocludder and either added or took away a doll. When the occluder was removed, infants saw either the expected number or an unexpected number of dolls. Representation of the transformation was tested through infants' increased looking at the unexpected outcome (see also Cohen & Marks, 2002; Feigenson, 2005; Simon, Hespos, & Rochat, 1995; Uller, Carey, Huntley-Fenner, & Klatt, 1999). Transformation of larger quantities has also been tested (McKrink & Wynn, 2004). These studies found that infants look longer to the unexpected outcome for small as well as large quantity transformations—small number transformation may be more precise whereas large number transformations are within the ratio limit for discrimination.

Ordinality judgments, similar to transformation tasks, require the discrimination of quantities and then the recognition of a relation between those quantities. Several studies have investigated very young infants' capacities for ordinal relations by habituating infants to a sequence of quantities such as ascending or descending amounts. Detection of the ordinal direction is implicated if, after habituation, a reversal of the sequence elicits an increase in looking. A list of relevant studies is in Table 6. An early study showed that infants could not successfully recognize ordinal relations (Cooper, 1984); however, later studies showed that if infants were presented with more information— for example, if they were presented three instead of two quantities in the sequence to compare or larger ratio distances among quantities— they could succeed (Brannon, 2002; Suanda, Tompson, & Brannon, 2008). Consistent with data from discrimination studies, this ability improves with age. In their studies, Suanda, Tompson and Brannon (2008) found that 9 month olds succeeded only when number and surface area were confounded (and thus discrimination need not depend on number per se) but 11 month olds recognized ordinal relations when surface area was controlled. Thus the older but not the younger infants' performances in this study are consistent with a mechanism that may be specific to discrete number judgments.

2.4 Number Abstraction: Infants' intermodal matching

Possibly the most compelling studies implicating an *abstract* numerical capacity are those that demonstrate infants' abilities to match quantities across modalities in looking while listening tasks— an ability that three year old children do only with great difficulty in tasks that explicitly ask them to match the number of visual and auditory events (Mix, Huttenlocher, & Levine, 1996). A list of the infant studies in this domain is provided in Table 7. Although most studies have tested visual to auditory intermodal matching, one has tested haptic to visual matching (Feron, Gentaz, & Streri, 2006). There are relatively few intermodal-matching studies and results are mixed, but this is an important class of experiments to consider because they may provide evidence of a capacity to apprehend, represent, and match discrete number independently of correlated modality-specific dimensions. In the first attempt to demonstrate this ability, Starkey, Spelke and Gelman (1983, 1990) used a preferential looking task in which infants could choose to look at one of two arrays of objects. They found that infants preferred to look at the visual array in which the number of elements matched the number of heard auditory events. For example, infants hearing 3 drumbeats preferred to look at 3 objects rather than 2. However, several attempts to replicate the study yielded the opposite result or no preference (Mix, Levine, & Huttenlocher, 1997; Moore, Benenson, Reznick, Peterson, & Kagan, 1987).

The mixed results and possible failure of infants in the previously described task have been attributed to the unnaturalness of the audio-visual pairings. Subsequent studies have thus attempted to test intermodal matching with more natural pairings. For example, Kobayashi, Hiraki, & Hasegawa (2005) showed infants dynamic scenes of puppets falling and making a noise when they impacted the stage. The impact sound corresponded to the number of puppets that fell to the floor of the stage and therefore presented a causal relation between the visual and audio stimuli. In their study, infants succeeded by looking longer to unexpected pairings. When infants heard three impact sounds but saw only two puppets, their looking increased. In a separate study, Jordan and Brannon (2006) tested infants' ability to match the number of voices to the number of people seen on a screen—also a more natural causal pairing. They found that infants preferred to look at the correct number of women corresponding to the number of voices being heard; infants that heard 2 voices preferentially looked at a display of 2 women whereas infants hearing 3 voices looked to the 3 women display. Recently, one study has shown an intermodal numerical capacity in newborns. Infants less than a day old showed preferential looking towards arrays of geometric figures that matched numerically to the number of syllables being heard in a word (Izard, Suan, Spelke, & Steri, 2009). Strong conclusions about whether, when, and how infants make intermodal matches in discrete quantities may not yet be warranted given that there are relatively few intermodal studies and that these have shown mixed replication success; however, this class of experimental design may ultimately provide the most convincing evidence for an abstract numeric capacity.

2.5 Summary

Overall, the results across the various tasks present a strong case for an infant cognitive system that responds in systematic ways to discrete quantities. Further, the data suggest that the processes that underlie these abilities share much with the adult system since infant behavioral responses show many of the same signature characteristics seen in adults and in other species. Whatever the eventual understanding of these abilities, the coherence of the phenomena indicate they are likely foundational to human numerical capacities.

3. Do we know less than we think we do?

When one apple is added to a set of two to make three, there is an increase in number; however, there is also an increase in surface area, the cumulative length of the contours of the objects, the overall weight and volume, and possibly the density of the items (for example, within the boundaries of a bowl). These other stimulus dimensions are not the same as discrete quantity, but they correlated and interdependent. Many of the earlier studies showing successful representation and discrimination of number by infants could be interpreted in terms of infant sensitivity to one of the other correlated dimensions. Starkey and Cooper's original 1980 experiment did not control continuous extent dimensions; surface area and number were confounded. When infants habituated to 2 dots and then dishabituated to the novel numerosity of 3, they were also dishabituating to a novel surface area; the study was therefore not unambiguous evidence for *number* discrimination. Among the arithmetic transformation studies, 5 of the 9 numerical transformations also allowed surface area or contour length to co-vary and thus be predictive of number differences, and most studies testing infants quantity knowledge of small sets in crawling or manual search procedures have allowed surface area, contour, and sometimes event duration to correlate with numerosity. These studies, although indicative of a quantity system that is sensitive to possibly many dimensions of magnitude, do not demonstrate a system strictly sensitive to discrete quantity. Of course they also do not show that such as system does not exist. However, a central theoretical idea underlying much of the current research on number discrimination is that there is a discrete number system that is distinct and independent from the processes that underlie judgments of continuous quantity (total amount) or general perceptual differences (configural patterns or texture). Contemporary researchers have thus sought to show that the mechanisms responsible for their experimental findings are specifically sensitive to number and they have tried to control for the possible influence of other correlated perceptual dimensions.

Accordingly in this section, we consider the problem of control: how successful have the contemporary efforts been in ruling out other dimensions? The section is long and detailed because— when viewed at low magnification— the consensus view of a distinct and specifically discrete quantity system holds. Close up, however, there are oddities that stand out against the overall coherence of the pattern, oddities that may not be noise but rather signals that the current consensus is missing a potentially important part of the developmental story. This section considers these problems and gaps, and in the final section of the review, we consider what an alternative developmental account might look like. The proposal we will offer is seriously under-determined by the extant data precisely because of the all-out effort to control— rather than to study – possible interactions among dimensions of discrete and continuous quantity.

3.1 Controlling and isolating dimensions

Changes in number are correlated with changes in other stimulus dimensions; thus, it is empirically difficult to know which dimension infants are attending to when discriminating numerically different sets. In a now classic study, Clearfield and Mix (1999) attempted to disentangle discrete number from some of these other variables. In their approach, they habituated infants to two correlated dimensions (number and contour length) and then used dishabituation responses to determine which dimension was the one infants were representing across the habituation trials. More specifically, they habituated 6-7 month old infants to 2 or 3 items—two-dimensional arrays of black squares on a white background—with constant cumulative contour lengths. Then, in testing, they showed infants two arrays: a novel numerosity with a familiar contour length and a familiar numerosity with a novel contour length. Clearfield and Mix reasoned that if infants attended to number, they should dishabituate to the novel numerosity, despite the fact that contour length had not changed. In the Clearfiled and Mix study, as well as subsequent studies, however, infants showed increased looking to changes in *continuous extent* but not to number (Clearfield & Mix, 1999, 2001; Feigenson, Carey, & Spelke, 2002; see also Experiment 1 of Cordes and Brannon, 2009 in which infants respond to both number and continuous extent), suggesting that they had attended to and represented the continuous quanitites in the arrays of dots and not discrete quantity.

One major point the field took from the Clearfield and Mix study was about methods. The study introduced a way to control and isolate dimensions of quantity. Since the publication of that study, researchers in the field have re-examined and re-designed experiments to test for sensitivity to number—controlling various dimensions such as surface area, contour length, and sound duration by holding these dimensions constant across habituation and testing. In such studies, the assumption is that if infants dishabituate or respond to a change in numerosity—

without the change in the controlled dimensions—infants must be doing so on the basis of number detection. Results from these studies, however, have complicated our understanding of infant quantity representation (see especially Clearfield and Mix 1999, 2001; Clearfield, 2004, 2005; Feigenson, Carey, & Spelke, 2002). This is because infants, at least sometimes, attend to and may even rely on these other dimensions when comparing arrays that differ in number. Of course, attention to these other dimensions does not mean that a number system does not exist. One potential interpretation is that the number system was not engaged in contexts and that infants used other dimensions to make successful discriminations. Another possible interpretation, however, is that the number system was engaged but that the system that determines discrete quantitity is *not* mechanistically *separate* from and not unaffected by other dimensions of quantity. That is, dimensions of continuous quantity may in fact play a very direct role in forming representations of discrete quantity.

The Clearfield and Mix (1999) result also led to another method of controlling for other dimensions— a method first introduced by Xu and Spelke (2000). Whereas Clearfield and Mix (1999) held all dimensions constant across habituation and testing, Xu and Spelke (2000) varied continuous extent dimensions during habituation and kept number constant. For example, infants in their original study were habituated to 8 dots that changed in surface area on each habituation trial. At testing infants saw displays of familiar numerosities (8 dots) and novel numerosities (16 dots) with item densities that were equal to one another and surface areas that fell within the range of areas already seen during the habituation phase trials. The control was interesting (as well as elegant and clever) for two reasons. First, because surface area was varied widely throughout habituation, the researchers argued that this could not be a predictable dimension for quantity representation. Second, because the item density was the same for both test displays, infants' preferences during testing could not be based on this dimensional difference between the two stimuli. Thus, an increased looking time to the novel number in this task should indicate detection of a discrete number change, and this is in fact what Xu and Spelke found in their study; but, as we will point out in the coming sections, studies using this method have led to some of the more unpredictable and difficult-to-resolve results in the literature. In brief, however, the two methods have different conceptual motivations; whereas the Clearfield and Mix approach measures which of two correlated dimensions the system attends to given some stimulus display, the Xu and Spelke approach measures competence: if all other possible solutions are removed from a task, can infants still process and represent discrete quantity?

Many studies since have used these two methods to control continuous extent dimensions and ask about attention to discrete quantity, and both types of controls are noted in the tables. Controls for visual procedures typically focus on surface area and cumulative contour—although some studies have attempted to control item density as well. For auditory studies, total duration, individual sound duration, and interstimulus intervals are among the variables that have been controlled. Of the 65 visual and auditory discrimination studies in Tables 1 and 2, 54 implemented continuous extent controls. Of these studies that implement controls, the majority used the Xu and Spelke method (44 studies); 8 used the Clearfield and Mix method and 2 papers included both methods of control (Fegienson, Carey, & Spelke, 2002; Clearfield, 2004). Researchers have therefore attempted to rule out reliance on at least some of the other cues as indicators to number in these studies. However, most studies have controlled for just one of the many correlated dimensions and most of the visual studies have focused on surface area controls. Even if findings from these studies did not lead to the oddities considered next, isolated controls for one or two dimensions (surface area or contour) are not sufficient to conclude that there is an early developed, internally stable representation system for discrete quantity (much less that it is abstract). The conclusion that the system responsible for these comparisons is specific to number and not influenced by continuous quantity dimensions requires the systematic study of possible interactions between number and the other co-varying dimensions in these stimulus arrays.

3.2 Small and large set discrimination

The first worrisome oddity derives from studies of small number comparison for which continuous extent was controlled. From the perspective of the consensus view, the evidence for *small number comparisons* by infants is unexpectedly weak. Across studies comparing small sets using various methods of habituation/familiarization (Tables 1 and 2), manual search (Table 3), crawling (Table 4), as well as transformation tasks (Table 5)— the majority of studies indicate that infants *do no*t discriminate 1 versus 2 without redundant continuous extent cues (Feigenson, Carey, Spelke, 2002; Xu, Spelke, Goddard, 2005; but see Feigenson, 2005) and they may not discriminate 2 versus 3 (Clearfield & Mix, 1999; Lipton & Spelke, 2004) without support of redundant dimensions. Across the nine experiments that have investigated 1 versus 2 quantity comparisons (Tables 1, 3, 4 and 5), seven studies have attempted to control continuous extent. Of these, only two showed behavior that suggests numerical discrimination (Feigenson, 2005; Feigenson & Carey, 2003) and only one study showed this discrimination in infants younger than 10 months (Feigenson, 2005; see also Starr, Libertus, & Brannon, 2013). In brief, documenting that young infants can discriminate sets sizes of 1 and 2 on the basis of number alone has been surprisingly difficult. We see this difficulty in documenting 1 versus 2 discriminations as a harbinger: at the very least, the consensus view is incomplete with respect to certain other potential factors that are relevant to infant performance in these tasks.

For comparisons of 2 versus 3, the results are somewhat better but still mixed. Across all the studies in the tables, there are at least 25 that directly investigated this comparison (see Tables 1, 2, 3, 4, and 7). In visual and auditory discrimination, five experiments indicated that infants discriminated without continuous extent cues (Bijeljac-babic, Bertonicici, & Mehler, 1993; Cordes & Brannon, 2009b; Starkey, Spelke, & Gelman, 1990; Strauss & Curtis, 1981; Wynn, 1996) and four studies suggested they did not (Clearfield, 2004; Clearfield & Mix, 1999, 2001; Lipton & Spelke, 2004), with one study yielding conflicting looking time and heart rate results (Brez & Colombo, 2011). Interestingly, *only one* published study to date that has controlled continuous extent has reported dishabituation or recovery of orienting time to novel numerosities in a 2 vs 3 comparison (Cordes & Brannon, 2009b)—the other studies demonstrated discrimination through preference for the novel over familiar quantity at test; although a habituation procedure was used, the infants did not dishabituate. However, in the Cordes & Brannon, 2009b study, infants dishabituated to *both* novel numerosity and cumulative contour length, suggesting that during familiarization infants were jointly attending to number and continuous extent; no study reports dishabituation to only the novel numerical quantity alone in a 2 vs 3 comparison. These findings might be attributed to the vagaries of testing infants and the looking measures, and thus as not contrary to the theoretical idea of an early and specific number system. However, as will propose later, these findings might signal a number system through which infants form discrete number representations by attending to multiple co-varying dimensions.

Among intermodal matching studies of 2 vs 3, three studies report discrimination (Feron, Gentaz, & Streri, 2006; Jordan & Brannon, 2006; Starkey, Spelke, & Gelman, 1990) and two report no discrimination (Mix, Levine, & Huttenlocher, 1997; Moore, Benenson, Reznick, Peterson, & Kagan, 1987). Of the intermodal preferential looking studies, two experiments report preference for the incorrect quantity, yet one of these studies uses these results to claim infant discrimination (Feron, Gentaz & Streri, 2006), whereas the other study interprets these results as evidence that infants are not using number alone (Moore et al., 1987). No manual search or crawling procedures have controlled continuous extent and demonstrated discrimination for 2 vs 3.

In our view, there are two potentially key findings in the just-reviewed studies of small-set discrimination. First, documentation of quantity comparisons of small numerosities of 1-3 items is difficult when there are not other correlated (that is, uncontrolled for) dimensions covarying with number. Second, and related, there is positive evidence that—when presented with arrays of items— infants attend to and may possibly rely on surface area, contour length, or (for auditory or event enumeration) total presentation and time duration. In addition, other studies that attempted to dissociate number from continuous extent variables showed discrimination only for continuous extent changes (Clearfield & Mix, 1999, 2001; Feigenson et al., 2002, although see also Feigenson, 2005) or equal attention to continuous extent and number (Experiment 1, Cordes & Brannon, 2009a). In sum, for small sets infants appear not to attend to number alone but use stimulus dimensions that *correlate* with number.

The contrast between the weak evidence for small-set discrimination and the robust evidence for large set discrimination by infants also seems relevant to a complete theory of number discrimination. The evidence for large number discrimination— evidence based on similar experimental tasks— has been readily documented across laboratories and investigators (see Tables 1 and 2). The evidence indicates that 6 month old infants reliably discriminate large quantities at a 1:2 ratio when certain continuous extent variables are controlled (in ways similar to those controls used for small number comparisons, Brannon, Abbot, & Lutz, 2004; Cordes & Brannon, 2009b; Wood & Spelke, 2005; Xu & Spelke, 2000; Xu et al., 2005), and this has been shown in both visual and auditory studies (Lipton & Spelke, 2003, 2004). There are still some questions as to whether the stimulus controls are adequate for concluding large set discrimination based strictly on discrete quantity (see Clearfield, 2005 and Mix, Huttenlocher, & Levine 2002), and, again, most studies only control for surface area and not other correlated dimensions. Nonetheless, within these limits, the consensus view of the lawful properties of the proposed number system are much better supported for the discrimination of large than small set sizes.

3.3 Procedures and dependent measures of discrimination

How problematic are these weaknesses in the data? The answer to this question depends, in part, on one's confidence in the behavioral measure of discrimination. Very young infants have few ways of demonstrating what they know; the principal measure in studies of young infants is looking behavior. Two different procedures and measures of looking behavior have been used to assess infant discrimination: 1) *dishabituation* to the novel quantity after habituation (or familiarization) or 2) *preference* for the novel quantity over the familiar one during the testing phase. Both measures are reported in the tables, however the measure reported as the main measure with reference to concluding the discrimination for each study has been bolded. Of the 30 reports concluding visual numerical discrimination, 16 used preferential looking as the main measure and 12 used dishabituation; 2 used both measures (Cordes & Brannon, 2009a; Jordan, Suanda, & Brannon, 2008).

Although both measures are acceptable in the field, frequently the two do not coincide even within the same study. In visual studies, infants sometimes dishabituate to a novel quantity but show no preference during the testing phase (e.g., Experiments 2 and 4, Cordes & Brannon, 2009a; Experiments 1 and 2, Cordes & Brannon, 2009b; Experiment 2, Feigenson, 2005); in other instances, infants show a preference during testing but no dishabituation (e.g., Experiment 1, Cordes & Brannon, 2008; Experiments 1 and 2, Feigenson, Carey, & Spelke, 2002; Experiment 3, Xu, Spelke, Goddard, 2005). Across all visual studies in Table 1, 43 have published relevant looking time information; of these, 16 have patterns for the two measures that suggest *different* conclusions about infant abilities. Of the visual studies, 6 of the18 that conclude numerical discrimination do not show clear dishabituation to the novel quantity (Brannon, Abbot & Lutz, 2004; Cordes & Brannon, 2008; Starkey, Spelke, & Gelman, 1990; Wood & Spelke, 2005; Wynn, 1996; Xu, Spelke, & Goddard, 2005). To further complicate matters, results from four visual studies indicate dishabituation to a novel quantity, yet researchers concluded there was no discrimination because preferential looking measures showed no difference—despite the fact that infants must be responding to *some dimension of novelty* from habituation to testing (Experiment 2b, Cordes & Brannon, 2008; Experiment 2 and 4, Cordes & Brannon, 2009a; Experiment 2, Wood & Spelke, 2005). Of the auditory studies in Table 2, only one of the four that concluded discrimination showed recovery of orienting or looking behavior (Experiment 2, vanMarle & Wynn, 2009).

It is not entirely clear which measure should be used in these studies—preference during testing or dishabituation/recovery of orienting behavior— however it is difficult to compare across experiments when varying measures have been used, when these measures have been shown to yield different results, and when researchers— faced with disagreement between the two measures— have interpreted the patterns in opposite ways. For example, results from Experiment 1 of Brannon, et al. (2004) showed no dishabituation to novel quantities in an 8 vs 16 quantity comparison; however, based on preferences in the testing phase, discrimination of the quantities was concluded. Cordes and Brannon (Experiment 2, 2009b) found the opposite pattern for the same numerical comparison—infants dishabituated but did not show preferential looking during testing— yet it was again concluded that infants discriminated the quantities.

Infant looking behavior is known to be a noisy and imperfect measure, with many problems (Aslin, 2007; Colombo & Mitchell, 2009; Gilmore & Thomas, 2002; Oakes, 2010; Rackin, Abrams, Barry, Bhatnagar, Clayton, Colombo, Coppola, Geyer, Glanzman, Marsland, McSweeney, Wilson, Wu, & Thompson, 2008); yet considerable progress has been made in a number of domains of infant perception and cognition using these procedures. Because it is extraordinarily easy to get null results in a study using looking time as the dependent measure, any statistically reliable effect seems to have meaning and replicated effects would seem to have considerable meaning. It is, thus, not surprising that researchers have not come to a consensus on which trials or how many should be averaged for statistical comparisons and have not come to an agreement about inclusion criteria for infants in the final sample: effects may be fragile and the parameters leading to the observed effects— as many may reasonably argue— might change or be a function of tasks, stimuli, or the ages of the infants being tested. It may, therefore, be difficult or even unreasonable in light of our test subjects and methods to define too stringently the appropriate measures or parameters. A defensible approach— and one that the field has taken— is to view results from a distance and to concentrate on the regularities obvious in the data across laboratories rather than quibble over differences in the measures chosen by the various researchers to support or negate infant capacities.

However, a closer look at the patterns across differing measures of looking behavior suggests regularities in this noise that may be signals to the underlying processes. One potential signal for theories in this noise is that infants, in number discrimination studies, often show discrimination by one measure *or* the other— but not both; infants do not systematically show *both* dishabituation or recovery behavior *as well as* a preference for the novel quantity during testing. Of the 24 visual and auditory studies that conclude numerical discrimination (and report sufficient data), only 9 show both a difference during the testing phase as well as recovery of head orienting or looking behavior. A potentially informative observation is the relation between the control method and the dishabituation results: dishabituation is observed when correlated dimensions are controlled by *holding them constant* across habituation and testing (the Clearfield and Mix control method). Twelve out of thirteen studies using this control (in Tables 1, 2, and 8) show this pattern. The one exception is the previously mentioned 2 vs 3 comparison in which dishabituation to both the novel and familiar quantities was found (Experiment 1, Cordes & Brannon, 2009b). When number or continuous extent is controlled *by varying dimensions throughout the procedure* (the Xu and Spelke control method), infants often fail to show dishabituation but may show preferential looking.

How might we understand this pattern? One hypothesis is that dishabituation depends on a strong internal representation of the dimension being tested (i.e., number) and that when other dimensions are held constant and *thus consistently correlated with number during the habituation phase*, infants build stronger memories of the information in the arrays. By hypothesis, these stronger memories for the instances they have seen may enable them to better discriminate *number* (even though the controlled dimension is also constant across habituation and dishabituation trials). However, when the arrays vary from trial to trial on various dimensions (the Xu & Spelke method), infants have to extract number in the face of broader stimulus differences and in so doing, we hypothesize, build weaker representations. Infants may be sensitive enough to discrete quantity as an isolated dimension and show this sensitivity in preferential looking measures, but robust representations— sufficient to yield robust dishabituation effects— may be formed only when multiple dimensions are correlated during habituation. At the least, the differences between the two measures— and the fact that they often do not coincide— suggest that infants may be processing the arrays differently when number is consistently correlated with another quantity dimension. These differences suggest that there is more to know about number processing than our current conclusion (that there is a discrete system controlling performance); at the very least, it would seem that numerical information interacts with co-varying dimensions of quantity. Investigating these interactions could lead us to an understanding of the seeming inconsistencies in the infant data.

At this juncture, it is worth noting that there are a few exceptions to this observed pattern that may be critical to forming a coherent picture of the early quantity representation system. Seven studies using the Xu and Spelke control have shown dishabituation to (only) the novel numerical quantity *and* a preference for the novel quantity during testing in infants younger than 7 months. These studies have an additional potentially telling commonality: four of them compared quantities at 1:4 ratio differences (Experiments 1 and 3 Cordes & Brannon 2009a, Experiment 3a, Cordes & Brannon, 2008; Experiment 5, Wood & Spelke, 2005a). We offer this conjecture: infants show discrimination by both measures when the differences were large because comparison in these cases depended less on precise representations of the specific discrete quantities. Intriguingly, two of the other studies showing both dishabituation and a preference for the novel quanitity at test differed from the majority of experiments in that they provided more information to the infants than is typical: one was a multimodal study in which *both* auditory and visual numerical information was provided for infants (Jordan, Suanda, & Brannon 2008) and one manipulated the timing of display presentations such that infants received a broader range of display arrangements (Wood & Spelke, 2005a). These exceptions, therefore, also point to the idea that the *strength* of a representation of a precise quantity may depend on correlations of among dimensions in the same way that one might expect it to depend on the number of trials. More specifically, if we assume that patterns in which infants show both dishabituation and preferential looking indicate *stronger* internal representations of discrete quantity, then the overall pattern suggests the following: first, infants' internal representations of quantity are often quite fragile, (perhaps for different reasons given different tasks and stimuli)—they are sufficient to support dishabituation or preference but often not both. Second, correlated dimensions lead to more robust (or more precise) representations such that discrimination of the old from new quantity is seen by both measures.

The hypothesis that follows is this: the nature of the evidence indicating discrimination will be dependent on an interaction between the presentation of redundant information and the magnitude of difference between the quantities being compared. To state it more clearly: we propose that when information about quantity is provided to infants through multiple dimensions and multiple modalities, the precision with which infants can represent precise *discrete quantity*[1] information over a series of arrays may increase. If this is correct, then the current research approach of trying to rule out any possible dependence on dimensions other than discrete quantity could be missing a critical part of the developmental story: how stimulus dimensions

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[1] Other dimensions such as surface area or contour may also be more precisely represented when various dimensions such as number are correlated with them; thus this hypothesis may not be specific to number per se.

correlated with discrete quantity also influence the engagement of the number system and support number processing mechanisms.

3. 4 What correlated dimensions might matter?

The extant evidence, though not conclusive, strongly implicates an early *sensitivity* to discrete number. In our view, the extant evidence also suggests that this early sensitivity is fragile and that infants are sensitive to many other properties of stimulus arrays that potentially interact with, influence, and may support attention to number. Several interesting hypotheses follow from this idea— hypotheses unlikely to be tested if the focus of research is on ruling out a role for other dimensions. For example, one possibility is that there are other dimensions that may be more salient and yield more robust representations than number itself; these other dimensions may not simply interfere with attention to number but play a role in the development of increasingly robust and precise discrete quantity representations. For example, because they covary with number, they could serve as cues that direct infants' attention to number. Within this context, the evidence on infants' unexpectedly weak discrimination of small set sizes, and the greater salience of other dimensions over number for small set sizes, might reflect their early reliance on readily detectable correlations among dimensions for small numbers. If these ideas have any merit, then we critically need a better understanding of infants' abilities to detect a variety of co-varying dimensions including surface area, cumulative contour, density, and size for visual sets as well as duration or interstimulus intervals for auditory and event comparisons.

There has been an increasing movement toward the study of other dimensions (e.g., Brannon, Lutz, & Cordes 2006; Brannon & Cordes, 2008; Clearfield, 2005; Cordes & Brannon, 2011; Hespos, Dora, Rips, & Christie, 2011). Table 8 lists studies that have directly investigated infants' sensitivity to continuous dimensions that correlate with the number of items. One finding is that— although surface area is discriminable— infants are not highly sensitive to this dimension and discrimination may require more than a 3 fold change for detection (when tested using methods that attempt to isolate this dimension, Brannon & Cordes, 2008). This result suggests that surface area is not be a likely candidate as a dimension that infants robustly find salient or as a dimension that either supports or interferes with attention to number. In this context, it is unfortunate that this is the one correlated dimension that has been most-widely controlled in studies of numerical discrimination while other dimensions still co-varied with number. Infants' attention to and discrimination of density and cumulative contour are arguably more relevant dimensions as research outside of the domain of number discrimination indicates that these are highly salient stimulus dimensions to infants (e.g., Karmel, 1969; Fantz & Fagan, 1975; Norcia, Pei, Bonneh, Hou, Sampath, & Pettet, 2005; Reith & Sireteanu, 1994; Salapatek, 1975; Haith, 1980) as well as relevant dimensions for adults in *explicit numerical* tasks (Allik & Tuulmets, 1991; Bevan & Turner, 1964; Gebuis & Reynvoet, 2011; Kreuger, 1972; Sophian & Chu, 2008 Stoianav & Zorzi, 2012). However, few studies have assessed infants' threshold of sensitivity to these dimensions (see Norcia, et al, 2005 and Norcia, Tyler, & Hamer, 1990 for related visual sensitivities) and even fewer studies have explicitly asked about sensitivity in relation to quantity representation (Clearfield, 2005).

One might suspect contour length to be a potent dimension in infant discrimination of arrays of objects as studies have shown that infants are attracted to and attend to visual edges (e.g., Bronson, 1991; Haith, 1980) and a very large literature in vision and neurobiology clearly indicate the existence of selective cells for detecting orientation of lines and edges (e.g., Burr, Morrone, & Spinelli, 1989; Mansfield, 1974; Pettigrew, Nikara, & Bishop, 1968;Schiller, Finlay, & Volman, 1976). Numerical studies have attempted to account for the variable of contour by controlling it (Cordes & Brannon, 2009a; Xu & Spelke, 2000), but only a few studies have directly investigated whether infants can use the dimension (Karmel, 1969; see also Banks & Ginsburg, 1980). The little evidence that exists (Clearfield, 2005; Clearfield & Mix, 1999, 2001) shows that contour length is discriminable at 1:2 ratios at the very least; however, infant ability to discrimate edge lengths at smaller ratio changes has not been studied. This is a critical limitation because researchers investigating infant numerical capacities frequently allow contour length to vary up to a 2:3 ratio difference from habituation to testing phases in procedures that presume continuous extent controls (e.g., Xu, Spelke, & Goddard, 2005). These ratio differences could well be discriminable by infants and possibly influential to infant performance in these tasks.

One set of studies may inadvertently provide evidence of contour length's importance to infant number discrimination. Brannon, Lutz, and Cordes (2006) showed that 6 month olds successfully discriminated a surface area that changed 2 fold. In a separate study also intended to test surface area discriminations, Brannon and her colleagues found slightly different results—infants failed to discriminate a surface area change of 3 fold (Experiment 2a, Cordes & Brannon, 2008). The authors have proposed that the differences in precision for surface area tracking may be because of the context in which infants were asked to represent area: in the first study, infants had to track the area of only one item that was held constant across the habituation phase. In the second study infants had to sum and represent the area across two or three items during the habituation phase; unsuccessful comparison in the second study is congruent with our previous conjecture that information is more readily extractable when multiple dimensions are correlated. There is, however, a further possible and compatible account for the results. In the first study, the 1:2 area ratio difference was accompanied by a cumulative contour length that changed 1.4

times from habituation to testing—a ratio difference of slightly less than 2:3. In the second study, the stimuli were such that there was *no contour length change* from habituation to testing (see Appendix A.1 for details). The findings are therefore potentially explainable by assuming that contour length is a more relevant dimension to infant discrimination than area and that infants are sensitive to quite small differences in this dimension (e.g., a 1.4 fold change). In another study, Brannon, Abbot and Lutz (Experiment 1, 2004) found that infants who were habituated to either 8 or 16 dots did not dishabituate to novel quantities (although infants did show a preference during testing—interpreted as discrimination and dishabituation approached significance). The lack of a robust dishabituation effect is somewhat surprising because other studies (Cordes & Brannon, 2009; Xu & Spelke, 2000) suggest these are readily discriminable numerosities for infants. Cumulative contour length, however, may provide an explanation: although number changed 2 fold from habituation to testing, the cumulative contour length only changed approximately 1.2 fold, which may be below what is robustly discriminable for infants (see Appendix A.2 for details). If contour length plays a role in infant representation of discrete quantity, the nondiscriminable differences in this dimension may have disrupted representation and discrimination of two typically readily discriminable quantities.

A further study suggesting a potential gap in understanding of the dimensions that matter come from another test of area discrimination with interesting infant behavior. Cordes and Brannon (Experiment 2b, 2008) habituated infants to a constant surface area while number varied across habituation trials. At testing, infants saw novel and familiar surface areas. Results from this test showed that infants dishabituated to the *familiar* surface area. This result seems peculiar; what would infants be responding to? The researchers suggest that infants may have noticed that there were novel numbers being presented at testing, however number was presumably controlled by being varied during habituation[2]. Estimation of the contour length change (given the provided information), however, suggests changes from the habituation to testing phase of up to 1.5 fold in cumulative contour length (see Appendix A.3 for details).

Our point here is not to argue for better or different controls with the purpose of showing that infants *can* discriminate numerical quantities without continuous extent cues. Nor are we arguing that cumulative contour is *the* factor driving discrimination. Rather, we highlight these odd patterns to emphasize the unexplored dimensions that may in fact be foundational to representation and discrimination in many of these tasks. *There are many under- and wholly unstudied dimensions* that correlate to number; surface area is only one, and many of the others may in fact be more relevant to the representation of discrete individuated items and to the apprehension of number (e.g., cumulative contour density, spatial frequency, visual spread). The field may be creating what seem like gaps and inconsistencies in measures and findings by not casting its nets wide enough and by not thinking about all stimulus dimensions that covary with and are statistically related to discrete quantity comparisons in infant experiences. In so doing, we may be missing the supporting role these dimensions play in the development of number concepts. The field has placed so much focus on trying to document sensitivity to number per se that we know very little about when or why infants attend to number, very little about infant sensitivity to other dimensions, and very little about how these dimensions may interact with or

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[2] Infants in this study were shown 10 and 15 dots in alternation during habituation and were tested with 7 and 21 dots with novel and familiar surface areas; number, as Cordes and Brannon proposed, should not have been a salient dimension in this procedure as it was varied during habituation. However, it is possible that because number only alternated between two quantities during habituation, infants may have habituated to the specific numerosities and thus noticed the change at testing in number—a suggestion made by Cordes and Brannon in the original publication.

engage the processes relevant to discrete quantity representation— despite evidence already latent in the data that suggest these dimensions indeed matter.

3.5 Summary

Infants show systematic patterns of numerical discrimination, but there are clear gaps and oddities— including the difficulty in documenting small number discrimination and the inconsistencies across the dependent measures of dishabituation and visual preferences. There is also the problem— in the service of attempts to document and isolate sensitivity to discrete number— of controlling for the many stimulus dimensions that correlate with the increased numerosity of a set of things. Unfortunately, we know very little about these other dimensions in their own right or infants' sensitivity to them. This is problematic because all these dimensions are inter-related and cannot all be controlled at once: increasing number while controlling for overall area requires making the individual items smaller, which changes cumulative contour as well as measures of density, leaving them open as possible detected changes at test. However, the relevance of these inter-related dimensions may not just be a question of finding the proper stimulus controls; rather, the relevance may be much deeper and pertinent to how we understand the number system and its development.

4. A New Set of Questions

To date, the field has constructed a coherent picture of infant capacities from the perspective of numerical biases; but there are inconsistencies in the data that would seem to lie very close to the core of what we think we understand about number: there is a long standing view that large number comparison is psychophysically lawful but not exact and small number comparison is more precise (e.g., Kafman, Lord, Reese, & Volkmann, 1949; Taves, 1941; Trick & Pylyshyn, 1994). The first troubling fact is that infants' small number discrimination is difficult to document. A second troubling fact is that, unlike any data or predictions from the literature on the adult numerical system, infants appear unable to compare small numbers to large numbers in many instances. This odd result could mean that there are two systems that do not communicate early in cognition (Cordes & Brannon, 2009a; Xu, 2000); however, it could also mean that we do not fully understand how infants represent these stimulus arrays— within the context of our methods and dependent measures as well as the relative influence and interaction of the various correlated dimensions.

Given the vagaries of infant research, it is seductive to conclude that the data mostly fit the concensus view and leave it there. However, science can only find answers to the questions that it bothers to ask. If the only question asked is whether infants can discriminate discrete quantities, with all other aspects of stimulus arrays viewed as irrelevant (and in need of control), then researchers cannot discover possible interactions with other continuous dimensions of quantity and may in fact not discover a more core principle underlying infant representation. Accordingly, we argue for a shift in theoretical question— a shift we believe is warranted by the gaps in the data with respect to the assumptions of the consensus account and also by the regularities in what many have assumed to be noise in the measures of infant looking behavior. We first consider some general possibilities about how discrete quantity judgments may interact with other co-varying dimensons and then offer a set of testable hypotheses within an alternative developmental framework that attempts to account for the odd results as well as the coherent findings that underlie the consensus view.

How might discrete quantity be related to other co-varying stimulus dimension?

By all accounts, number discrimination requires that the perceptual system detect an array made up of individual elements. An array with elements that are too crowded, that are presented too briefly, or that are presented with contrast that is insufficient to enable segmentation of the items may not be processed as a set of discrete elements. At the limits of sensitivity, other dimensions will certainly matter to number judgements and these limits may change with development and may change differently for various stimulus properties relevant to the extraction of discrete quantity. Within this view, then, the study of infant sensivity to the stimulus dimensions that enable the extraction of discrete quantity is relevant to a complete theory of the development of numerical perception and representation. These ideas are in line with the current concensus view and should not be points of controvery in the field. Reserachers investigating infant numerical capacities would likely agree that there are factors that will influence what is perceived as discrete versus not discrete and that these factors will change quantity representation.

Researchers likely also recognize that the perceptual system extracts other classes of properties about perceptual arrays, including overall spatial extent, density, the entropy in the configuration, and orientation of items. These properties could be independent and distinct from the extraction of discrete quantity; however, because these dimensions co-vary with one another, it might be expected that perceivers learn the statistical relations among these dimensions and use knowledge of their correlations when comparing sets. For example, density may be used to compare sets when density information is particularly salient or easier to extract than discrete quantity. Moverover, because sensitivity to these various properites may develop at different rates and because some may be more readily extracted in some experimental contexts than others, the dimensions influencing performance could be differentially weighted in different tasks. By one view, these proposals are not in opposition to the concensus view; judgment of discrete quantity may not be directly influenced by these other dimensions, although these co-varying dimensions may influence attention to discrete quantity and thus the engagement of the system. By a second view, this idea may in fact counter the current theory, however: the number system might operate on information from these covarying dimensions to determine discrete quantity. Either of these views would mean that performance in number judgement tasks may not always be based on the same information (see Gebius & Reynoevet, 2011 for adult data supporting this idea) and experimental research needs to do more than control for this information—we need to study it.

A long history of research on the definition and independence of perceptual dimensions (e.g., Lockhead, 1972; Garner, 1974; Jones & Goldstone, 2011) suggests grounds to consider the second alternative— that the stimulus dimensions that co-vary with number actually play a role in the determination of *discrete* quantity, and a role that may change with development. This research on defining perceptual dimensions begins with the fact that for any stimulus there is an infinite number of objectively correct descriptions of the information available and often multiple psychological descriptions (Garner, 1974; Smith & Kemler, 1978). As Garner (1974) warned, the experimenter-defined dimensions in any task may not be the psychological ones that are used by the subject in that task or those that determine behavior. More critically, the extant research suggests fundamental limits to the decomposition of perceptual experience into psychologically independent dimensions (that is, into dimensions that are unaffected by covariation on other dimensions, Beals, Krantz & Tversky, 1968; Garner, 1974; Jones & Goldstone, 2011). Our phychological experience and categorization of color provides one example. Color is composed of three mathematical degrees of freedom, which may be identified in terms of hue, saturation, and lightness. However, our intuitions (and language) treat color as a single dimension, and perceivers— without extensive training— cannot decompose color into those subdimensions (Garner & Felfoldy, 1970; Smith & Kemler, 1978). Stimulus spaces that are physically multidimensional like color typically have dimensions that are characterized as *integral*—perceptually non-independent and interactive such that judgements on one are influenced by the values of the other (see Jones & Goldstone, 2011, for a recent discussion, and also Beal, Krantz & Tversky, 1968; Attneave, 1950; Shepard, 1964; Nelson, 1993). Number could be a separate independent dimension—one to which attention may be shifted towards or away given the specifics of the task. But because number resides in a complex multidimensional space of physically covarying dimensions, it may— as is the case with other such perceptual spaces— be more integral than separable.

For many multi-dimensional spaces, there is a general developmental trend toward increasing separability of dimensions (e.g., Smith & Kemler, 1977; 1978; Schepp & Barrett, 1991; Treiman & Breaux, 1982; Ward 1980; 1983), and research shows that experience and training may foster the extraction of dimensions (Jones & Goldstone, 2011; Nelson, 1993) and the formation of of new perceptual dimensions (e.g., Goldstone, 1998; Goldstone & Styvers, 2001; Jones & Goldstone, 2011). We suggest that infant research, in a broad sense, should aim to understand what determines the salience and separability of dimensions within this complex multidimensional space.

Signal Clarity: a proposal for infant quantity representation

With these fundmental questions about the relevant stimulus properties for number perception still unanswered, we offer the Signal Clarity hypothesis as a guide to pursuing the empirical oddities in the infant number discrimination literature and their potential meaning. We begin with the assumption, that all stimulus arrays have potentially many perceptual descriptions. When infants are presented with an array of items potentially describable by an internal cognitive system in terms of the shapes of the items, their color, their continuous extent, density, contour length, and number, the infant perceptual system must select (or settle on) a particular description. Further, because many of these tasks require that infants select a description aross *a series* of stimulus arrays (such as in an habituation procedure), the stimulus description is likely to depend on the particular *regularities across those arrays or events*. This means that within a specific task, what is settled in on as *the* representation of the series will depend on at least two factors: 1) the psychological description of each array— given what is readily detectable and extractable by the perceiver's system, and 2) the accumulation of occurrences of descriptions across the series of exposures. The clarity of the signal for any given dimension—clarity that will lead to a representation that is strong or fragile— will depend on these two factors. If there is a dedicated discrete number system as is implicated by the literature, that system— whether the information it extracts is strictly independent of other dimensions or potentially interdependent— has to be engaged in the task; the infant's system has to "know" that the task— and thus the right stimulus description and representation of stimuli arrays— is about number (see Brez & Colombo, 2012 for results consistent with this idea).

One potential source of the gaps and problems in the infant data may be the experimental and stimulus contexts in which the clarity of the signal for number (as either an independent or inter-dependent combination of integral dimensions) is weak. Certainly, if arrays are not viewed as discrete items (for example, if contrast is low with dark grey dots on a medium grey background), or if other perceptual stimulus signals are salient (if arrays are made of novel and very interesting individual objects or if one but not all of those objects moved), infants might not show their optimal abilities in representing and discriminating numerosities. If number is in fact a separable dimension, those dimensions that regulary covary with number such as contour length, density, and spatial extent might be particularly salient dimensions relative to the signal for number and therefore be even more distracting or interfering with the formation of robust representations of number. However, unlike the dimensions of contrast, color, or item kind, the dimensions of density, contour length, and spatial extent are potentially smart cues for the cognitive system to attend to for determing discrete quantity as they are predictive of the relevance of number (and if integral to number might also interact in potentially helpful ways to the perception of number itself). Thus, we propose that "attention" to these co-varying dimensions supports attention to number— both within a short experimental session as well as across experiences in the real world.

We offer the Signal Clarity hypothesis—the idea that the fundamental problem for infants is that they must discover that discrete quantity is the relevant task dimension and that stimulus properties that make number more perceptually salient are key to performance— as a general framework for future research. The framework leads to five testable predictions.

Hypothesis 1: The Weber Fraction is malleable (and depends on the factors influencing signal clarity) Infant detection of a visual stimulus is highly influenced by contrast and spatial frequency (see Banks & Ginsburg, 1980). The psychological description of a stimulus will necessarily depend on these limits and influences. At the very least, the ability of the infant system to detect and accurately describe a stimulus in terms of number will be influenced by the clarity of the signal as influenced by these same factors. Thus, the Signal Clarity hypothesis predicts that contrast and density (or spatial frequency and crowding) will influence representation and comparison of quantity. Numerosities presented in arrays of high contrast and low densities are more readily detected and more robustly represented than the same numerosities presented at low contrast and high density. If this is correct, then the Weber fraction should not be fixed but rather will be *moveable*— an idea already supported by literature on intermodal reduandancy (e.g., Jordan, Suanda, & Brannon, 2008), but extended here to basic sensory properties that influence detection of the items in array (for related ideas and data see also Lorenco & Longo, 2011; Suanda, Thompson, & Brannon, 2008). Within this framework, infant failure and success in discriminating quantities may not be best described as solely "ratio dependent" since that ratio will depend on stimulus factors.

Hypothesis 2: Redundant dimensions support robust (and more precise) representations. By hypothesis, signal clarity depends on the amount of noise presented with the stimulus and also on the frequency and duration of exposure to the signal as it is detected by the infants' sensory system. The theoretically relevant aspects of noise in infant habituation studies may be related to the variation across experienced arrays in the dimensions the system might be trying to extract (e.g., color, shape, number, cumulativce contour, surface area). When a dimension, for example number, is presented *dissociated* from other dimensions, such that those other stimulus dimensions vary widely across exposures, there is both good evidence for a relevant constant dimension but also potential difficulties in finding that signal, or any signal, in so much variation. The representation with respect to any of the dimensions may be weak. This means that the infant system may benefit from reduced variability— from *invariance and redundancy along several* dimensions. Consistent with this idea are the data reviewed here showing that when infant tasks present a series of number arrays with dimensions held invariant (in a Clearfield and Mix procedure) infants show more robust representation of both number *and* other dimensions (Brannon, Lutz, & Cordes, 2006; Clearfield & Mix, 1999; 2001; Clearfield, 2005; Cordes & Brannon, 2009b; Cordes & Brannon, 2008). Hypothesis 2, however, is a general hypothesis and concerns variability in general— in the colors, the shapes, the background, as well in the co-varying dimensions of continuous quantity. For example, most (but not all, Brez & Columbo, 2011; Feigenson, 2005; Strauss & Curtis, 1980) studies of number discrimination use arrays of dots or squares, with color and shape held constant both within and across arrays, which in our view supports more presise representation of both the number, the shape, and the color. We predict better discrimination of number when dimensions such as shape and color are constant across arrays rather than varied. That is, orthogonal variation of any kind— varying color, shape, item size, or contour, for example— could be harmful to infant number discrimination. The key prediction is that the Weber fraction itself will vary across these testing conditions, and the further question is whether it does so less when the varying dimension is physically separable from number (e.g., shape or color) versus when the varying dimension is physically integral with number (e.g., density or contour length)—a prediction set forth in Hypothesis 3.

Hypothesis 3: Physically correlated (or integral) dimensions support number discrimination

We propose that the dimensions of continuous quantity that physically co-vary with number and that are most problematic for empirically demonstrating a sensitivity to number may play a special role in determining numerical signal clarity early in development. At the very least, these dimensions may be useful predictors about the relevance of number. That is, increased density, increased continuous extent, and increased contour length are imperfect but potentially useful indicators that there may have been an increase in the number of items in an array; conversely, decreased density, decreased continuous extent, and a decrease in contour are potentially useful indicators that there has been a decrease in the number of items in a scene. However, it is also possible that number perception at the sensory and perceptual level is like color— a unitary psychological dimension that resides in multidimensional space (see Jones & Goldstone, 2012, for insights into the geometry of such dimensional spaces). From both the weaker and stronger hypothesis, the nature of the co-variation with number on these stimulus dimensions should matter to perceptual number judgements. When these continuous quantity indicators of number vary randomly or orthogonally with number (as in the Xu & Spelke control mehod) extraction of number should be more difficult than when they are held constant (as in the Clearfield & Mix control method). More specifically, we hypothesize that when these other dimensions are held constant throughout the presentation of a specific numerosity during habituation, the precision with which infants represent the number (as a clear and single point in the stimulus space) will be more robust and therefore more precisely discriminated from other numerosities. We also offer this conjecture: the current data that suggest a "divide" between large and small sets may in fact be explained by non-redundant dimensions (or the lack of redundancy in the dimensions that compose the infant system's description of the stimuli). Successful comparison may result when dimensions are redundant rather than varied. Variation along any dimension might be harmful to numerical represenation, but if number is composed of integral dimensions (or, at the very least, if number is highly correlated to certain dimensions more than others), then variation of such dimensions (contour, density, area) will influence repesentation more than variation of color or shape.

Hypothesis 4. There are privileged axes of change within a multi-dimensional number space. Previous research on integral-dimensional spaces indicates that there are often priviledged directions of change (Smith & Kemler, 1978; Foard & Kemler, 1984; Grau & Kemler Nelson, 1988; Melara, Marks & Potts, 1993; Jones & Goldstone, 2012). We predict that there will be priviledged directions of change within the multi-dimensional space of stimulus properties that vary with number. In particular, we predict that the infant system will be aided when changes in dimensions are correlated in one way (larger number and denser stimulus array for example) than in the other way (larger number and less dense items in the array). That is, when a change in density, for example, is positively correlated with numerosity differences, infants will be better able to latch onto number, and infants will more robustly represent and discriminate these discrete quantities at smaller ratio differences. However, when these continuous quantity indicators are negatively correlated with numerosity differences— they will hinder the detection, representation, and discrimination of discrete quantity. We do not specifically predict which dirrections of change in this n-dimensional space will be privileged, nor is it yet clear which direction for each dimension will correspond to the directional change of another dimension (e.g., density and total area may go in the same or opposite directions), but the determination of the existence of directions of change that lead to finer discriminations is critical to understanding the number system and its perhaps multidimensional foundations.

Hypothesis 5: A developmental trend from more integral to more separable dimensions of quantity. Beals, Krantz, & Tversky (1968) defined independent dimensions as those that were perceptually isolated in the sense that judgements on one dimension were not influenced by a variation on the other. For example, discrimination of two squares of a particular lightness should not depend on whether those two squares are 1 inch or 3 inches in size. Almost all pairing of dimensions— both integral and separable— fail this definition and thus the critierion of complete independence of number judgements from other dimensions is likely too stringent for the definition of a distinct system that represents numerosity. Garner (1974) distinguished integral and separable dimensions not by complete independence but by the degree to which a perceiver could selectively attend (and thus represent) values on a single dimension unaffected by variation on other dimensions. Garner used a suite of selective attention tasks to operationally define integral and separable dimensions, with selective attention to integral dimensions being difficult if not impossible and selective attention to separable dimensions occurring more spontaneously and readily. Across the many dimensional combinations that were examined within this framework, dimensions that were not physically independent (such as saturation and brightness of a color, or pitch and timbre of a sound) tended to be more integral; selective attention to one dimension, unaffected by variation in the other, was more difficult (see Garner, 1974). Although many dimensions show some degree of integrality in adulthood, a large literature indicates that children may in fact show greater difficulty in separating dimensions, possibly perceiving stimuli dimensions as more integral than adults (e.g., Smith & Kemler, 1977; 1978; Shepp & Barrett, 1991; Smith & Kemler; Treiman & Breaux, 1982; Ward 1980; 1983). Ward (1980; 1983), using one of Garner's defining tasks, examined the integrality-separability of number, extent, and density of dot arrays for preschool children and adults. Whereas adult judments suggested that these three dimensions were perceptually separable, and could be judged independently, the evidence from preschoolers fit the definition of integral dimensions. Accordingly, we predict that number and the co-varying dimensions of continuous quantity will become increasingly separable *with development*.

These hypotheses— and the ideas and open questions that underlie them— are not presented in oppostition to the consensus view that infants represent discrete quantity or that their number system may possess many of the same core properties as the adult system. The hypotheses and larger ideas do, however, question the idea that that number system is a higher order abstract system from the onset in early infancy or that the perceptual factors of an array are only relevant in so much as they "allow for encoding of number." We suggest instead that a system that outputs an estimate of discrete quantity is responsive and in fact composed of sensitivies to a variety of stimulus properties, and especially those that physically co-vary with number. Further, we argue that describing numerical abilities as the result of a number system that "uses" perceptual factors to encode discrete quantity may not be a useful model for understanding development of numerical reasoning or the inconsistencies in the data. Proposing that there is a numerical system with particular traits and going no further than that is a very broad *description*—not an explanation of what is occurring in the cognitive system— and may in fact cause our field to overlook the bigger questions that will unify our understanding of numerical representation with the rest of cognitive development and lead to a deeper understanding of quantitative reasoning and its development over the life span. We offer the hypotheses and ideas—although they are not yet supported by data—as a path to better understanding the full pattern of findings in the literature. These ideas broaden the research agenda. Instead of ruling out a a role for other dimensions and taking a binary "yes-no" measures of discrimination, the Signal Clarity hypothesis seeks a broader and more nuanced understanding of the perceptual dimensions of number in relation to the complex stimulus space that characterizes arrays, and asks how the perception and discrimination of numerosity may benefit (as well as perhaps be limited) by this stimulus structure. Based on the extant data on infant number discrimination, we propose that Signal Clarity— the unambiguity of the relevance of changes in number in this complex stimulus space— may be supported by the covariation of the dimensions of continuous quantity that are integral to variation in number and that development proceeeds from more to less dependence on these integral dimensions in representing discrete quantity.

Conclusion

A considerably large body or research conducted over the past 30 years has greatly contributed to our understanding of the early cognitive system's sensitivity to number. Results from these studies have led to interpretations that, from a distance, are coherent; however, the peculiarities and odd findings in this large body of data are probably not best left ignored and tackling them may take us to a deeper understanding of number perception. We offer the Signal Clarity proposal, and the five testable hypotheses, as useful directions toward that more complete understanding.

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Appendix A

1. Brannon, Lutz and Cordes (2005) tested infants in a 2 fold surface area change. They familiarized infants to one item (an Elmo face) with a surface area of 43.6 cm^2. At testing infants showed differential looking to an Elmo face half the size (21.8 cm^2). This represented a 2 fold change in total surface area (1:2 ratio difference). The exact contour length of the Elmo face is not reported, however, if it may be approximated as a circle, then contour lengths were approximately 23.41 cm and 16.5 cm. The difference between the two is a 1.4 fold change, meaning contour length changed 1.4 fold from habituation to testing.

Cordes and Brannon (2008) tested infants on a 3 fold surface area change. They familiarized infants to displays of dots with an overall surface area of 50 cm^2; this surface area was constant over arrays of 2 and 3 dots that alternated in the familiarization trials. Contour lengths in familiarization thus alternated between a total of 43.41 cm and 35.4 cm. At test, infants saw displays of novel and familiar surface areas. The novel surface area was 150 cm^2—a 3 fold change. Because this was contained within one dot, however, the contour length was 43.41 cm. Thus the contour length did not change. This lack of contour length change may have yielded the result of no dishabitaution that Cordes and Brannon (2008) found. Important to note is that calculations for all possible stimuli in the Cordes and Brannon (2008) study actually predict that infants habituated to the large surface area should not dishabituate to any test stimuli while infants habituated to the small surface area should not dishabitate to the novel area, but might dishabitate to familiar.

Of interest is that Cordes and Brannon (2008) also tested infants in a 4 fold change in area from 50 cm^2 to 200 cm^2 (a 4 fold change). The contour of the displays for this study (in one condition) were 70.90 cm and 86.83 cm during habituation. At testing the novel surface area display had a contour length of 25.07 cm. The contour length change was therefore more than 2 fold from either habituation display to testing—predicting the observed dishabituation.

2. Brannon, Abbott and Lutz (2004) habituated infants to displays that ranged in contour length between either 26.6 cm – 59.49 cm or 37.62 cm- 84.13 cm (for 8 or 16 dots respectively, depending on the habituation condition). The average cumulative circumference was 45.12 cm for 8 dot displays and 63.92 cm for 16 dot displays. At test all infants saw displays with total contour lengths of 37.73 cm and 75.46 cm. Calculating the nearest distances in contour lengths from habituation displays to novel display test trials, there was a 1.3 fold change for infants habituated to 8 dots and effectively no change for infants habituated to 16 dots. The 1.3 fold change accompanied by a no-change contour group may have resulted in the results of non-dishabituation to novel quantities when the orders were collapsed.

It should be noted that there are other possible ways to compute the distance between the cumulative contour lengths of the habituation displays and those of the test trials. Another calculation could be computed by taking the average of all the habituation displays and calculating the distance to the testing stimuli dimensions. Doing so yields slightly different results.

3. In this experiment, infants were habituated to a constant area that was spread over arrays of 10 and 15 dots. Infants were either habituated to a large surface area (150 cm^2) or a small surface area (50 cm^2). In habituation, the small surface area group saw displays that alternated in total contour length between 79.2 cm and 97.05 cm. Infants in the large surface area group were habituated to alternating displays of 137.29 cm and 168.135 cm. At test, all infants saw displays

of 7 and 21 dots, with 50 cm^2 and 150 cm^2 total surface area respectively. The cumulative contour length for these test displays were 63.30 cm and 198.9 cm. Contour length thus changed for familiar areas between 1.1 and 1.5. If a 1.4 change is necessary, these changes in some instances may have been sufficient to yield dishabituation results across the group of infants.